Assessment of Reaeration Equations for River Tungabhadra, Karnataka, India and Generation of the Refined Equation

S. Ranjith¹, Anand. V. Shivapur¹, P. Shiva Keshava Kumar² and Chandrashekararaya. G. Hiremath³

¹VTU-PG Studies, Belagavi-590018, India.
²Department of Civil Engineering, PDIT Engineering College Hosapete-583201, India.
³Department of Water and Land Management, VTU, Belagavi-590018, India.

Authors’ contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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ABSTRACT

The ability of a water body to self-purify itself is dependent on its re-aeration rate ($K_a$). This rate is necessary to calculate the dissolved oxygen content in the waterbody. This rate also depends on some variables that include the stream velocity, stream bed slope, cross section area, water depth, frictional velocity, discharge rate, Froude’s number and a number of other things. For the purpose of this study, thirteen empirical equations are considered when evaluating the performance of the re-aeration rates. This is done with respect to the size of the Tungabhadra river. Observation of the re-aeration rate for this study was done using mass balance approach. The data needed for this was gotten from field investigation data obtained from 288 separate samples (6 different sites) between the period March, 2017 to December, 2018. The performance evaluation of the re-aeration equation was done via the implementation of least square techniques. The following statistical error...
methods were applied in due course; standard error, normal mean method and mean multiplicative method. The results of the methods are 0.16, -0.0006 and 2.75. The coefficient of correlation for this was 0.91 and by interpretation, it shows an efficient outcome.

Keywords: Dissolved oxygen; DOBT; reaeration coefficient; Tungabhadra River.

Orcid I'd: https://orcid.org/0000-0002-5342-9842

1. INTRODUCTION

The mechanism of dissolved oxygen transfer through internal turbulence and mixing has gained much attention in recent years attributing to further study and investigations. Reaeration is the process of physical transfer of oxygen from the atmosphere into the water body, when the concentration of dissolved oxygen goes down with respect to saturation Dissolved oxygen of the stream at a given temperature. In 1925 Streeter and Phelps [1] stated that "Rate of absorption of oxygen is directly proportional to a dissolved oxygen deficit." In 2005 Jain and Jha [2] carried out research work on sensitivity analysis between Dissolved oxygen and Reaeration rate ($K_a$) whereby they concluded that a small change in $K_a$ gives a larger gap in the Dissolved oxygen. Hence $K_a$ plays a very important role to keep up (maintain) healthy ecosystem of the stream.

Gases gets transfer into water bodies from atmosphere. There are two theories which explain widely for both surface and estuaries water bodies [2-5]. First theory explains only about the standing water and second theory which gives information about the running water. Two film resistance theory assumes that substance moving in a layer by layer form develops maximum resistance between these two layers where the transfer of natural gas takes place. Second theory, i.e. surface renewal model, which assumes stream consisting of layers of water and when these layers are brought to the surface for a period of time, air exchange takes place. As these layers move away from the surface, they mix with the bulk liquid. Prior to the situation envisioned by two-film theory, the dissolved gas penetrates the film, and hence, it is dubbed as penetration theory. In 1951 P.V Danckwerts [6] altered the elegance by argumence that the liquid elements reach and leave the interface arbitrarily and their contact is designated by a statistical delivery. This approach is labeled as the surface renewal theory.

The General Governing equation for oxygen transfer can be written as

$$ V \frac{dc}{dt} = K_a (C_s - C) $$

Where $A_a$ is the surface area of water body ($m^2$), $V$ is the volume of water body ($m^3$), $K_1$ is the mass transfer velocity in liquid laminar layer ($md^{-1}$), $C$ is the oxygen concentration in water (mgL$^{-1}$) and $C_s$ is the saturation concentration of oxygen (mg L$^{-1}$).

In cases where the air-water interface is not constricted, the volume is $V = A_a H$, where $H$ is the mean depth (m). Thus, Equation (1) is expressed as

$$ \frac{dc}{dt} = K_s (C_s - C) $$

where $K_s$ is the re-aeration rate coefficient (d-1), which is equivalent to $K_a = k_1/H$.

Above equations deliver idea into how the mechanism of $K_a$ operates. The direction and scale of the mass transfer depend on the difference between the saturation value and the actual value of dissolved oxygen concentration in the water. Oxygen re-aeration rate can be induced to different temperatures.

$$ K_s (T) = K_{s0} \theta^{T-20} $$

where, $\theta = 1.024$ for pure water. In the rest of this paper, $K_s (20^0)$ is used as $K_s$.

Many researchers have developed/predicted reaeration equation for both standing and running water. In that 1925 Streeter and Phelps [1] developed a water quality model which became the bible for all researcher to development reaeration equation. William and Connorm developed empirical equation re-aeration $K_a$ in 1956 based on surface renewal concept which depends on dissolved oxygen balance technique [8,9,10], distribution equilibrium concept [11,12] and tracer method [13] according to researcher $K_a$ are directly proportional to velocity and depth of water. Churchill [9], Dobbins [14] and Streeter and Phelps [1] carried work on experimental esteems. Edward [15], Moog[22] worked on equilibrium distributed technique. Tesivogal and Kernel [19] developed an empirical equation based on tracer techniques. Several empirical
equations developed by using different method but all are related to stream variable such as bed gradient, wetted perimeter, flow velocity, depth of waterway, Froude number, shear stress.

**Study area:** In this case study we have selected River Tungabhadra which flows through Harihara taluk, Davanagere district, Karnataka, India. In this region stream is heavily polluted by industrial activity and domestic waste discharge at downstream side at Harihara. Tunga and bhadra river are tributaries of Tungabhadra formed by confluence at Kooli at altitude of 610mt above MSL and Tungabhadra travelling along Karnataka and Andhra Pradesh and finally join in River Krishna. Harihara region fall under semi-arid condition in which moderate to higher summer with erratic rainfall and moderate winter with erratic rainfall.

For the present study three villages which are located in the downstream side of Harihara town, namely Nalawagal, Nadhiralahalli, Airani were selected. These places have some socio economic and industrial. The municipal water from all these villages are places are directly discharged into stream and Harihara Poly Fiber which produce rayon grade pulp discharge approximately 30000/- liter per day and Rayon industry which discharge 10000/- liter per day [27]. Both industries are located in left bank of river Tungabhadra near Kumarapattanam. Selection of sampling station was done based on maximum mixing of effluent along width and depth of stream taken. Segment of sampling stations are in shown above Fig. 1.

**Calculation of Reaeration constant via DOBT method:** As presented in the Fig. 2, the domestic wastewater and industrial effluent are discharged at the segment 2 (just downstream of Harihara taluk) of the stretch. Right at the downstream of the waste discharge point, stretch is subjected to higher BOD concentration. At this point the DO of the stretch exhibits the exponential depletion attributing to the microbial activity in degrading organic matter. Further downstream of the stream, the lower removal rates happen as the more hard-headed natural matter debases at a slower rate and finally Biochemical oxygen demand decay model is shown in equation (4).

\[
L = L_0 e^{kr(x/U)}
\]  

(4)

Streeter and Phelps [1] model derived for a point source of BOD is given by:

\[
D = D_0 e^{[ka(x/U)+k_dU/(K_s+K_d)]} e^{[k_r(x/U)-e^{ka(x/U)}]}
\]  

(5)

where \(D=U/S\) Dissolved oxygen deficit mg.L\(^{-1}\), \(X=\text{distance travelled between } U/S \text{ and } D/S\) in Km, \(U=\text{stream velocity Kms/d, } X/U=\text{travel time}\), \(L=\text{BOD concentration (mg.L}^{-1}\), \(L_0=\text{initial BOD Concentration (mg.L}^{-1}\), \(K_r=\text{BOD loss rate (d}^{-1}\), \(K_d=\text{deoxygenation constant in stream (d}^{-1}\), \(K_s=\text{settling removal rate (d}^{-1}\), \(K_e=\text{reaeration constant (d}^{-1}\).

For calculation of kinematics constant such as BOD decay rate, Deoxygenation rate, and Reaeration rate, we have selected stretch between 19.5 km to 33 km i.e., downstream of Harihara to Airani village. In this region we are receiving industrial effluent from Harihara poly fiber and grasim rayon industry along with domestic waste discharge into stream in Nalawagal, Nadhiralahalli and Airani and there is no other source of discharge and obstruction to stream water. We have selected six sampling stations based on maximum mixing of effluent along width and depth of stream water taken.

According to the outline in Equation (4), the BOD decay model outline was created by plotting In L against (x/U), and it resulted in a straight line with the slope of kr. [25]. For the purpose of this project, the computed average of the higher values of \(k\) in the initial stretch was calculated as the BOD loss rate. On the other hand, \(K_e\) and the computed average of lower values of \(k\) while journeying further downstream was calculated as the de-oxygenation constant \(K_d\). DOBT was used to determine the reaeration constant \(ka\), and the DO mass balance equation (Equation (5)) was used for the computations. The method requires the measurement of all the sources and sinks of DO. Only the reaeration values will not be measured. After that, \(K_e\) is computed as the difference in the reaeration required to achieve the DO concentration that was measured at the downstream end of the stream segment under investigation. It is evident that this technique is equally effective when compared with other distributed equilibrium methods and tracer techniques. The light and dark bottle method was applied to establish the rates of respiration and photosynthesis. The results show that the river has a minimal amount of algae matter which could not lead to substantial variation in DO due to respiration and photosynthesis. After measuring all the variables were except \(K_e\) using the least square was then applied to determine the estimated value of \(K_e\) for each data set.
Table 1. Some frequently of used predictive re-aeration equation

<table>
<thead>
<tr>
<th>Investigator</th>
<th>Empirical formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Connor and William Dobbins [14]</td>
<td>$K_a = 3.901 U^{0.5} H^{1.0}$</td>
</tr>
<tr>
<td>Churchill et al. [9]</td>
<td>$K_a = 5.010 U^{0.969} H^{-1.5}$</td>
</tr>
<tr>
<td>Orlob &amp; Krenkel et al. [15]</td>
<td>$K_a = 173 (SU)^{0.404} H^{-0.66}$</td>
</tr>
<tr>
<td>Owens et al. [16]</td>
<td>$K_a = 5.35 U^{0.67} H^{-1.85}$</td>
</tr>
<tr>
<td>Langbein and Durum [17]</td>
<td>$K_a = 5.14 U H^{-1.33}$</td>
</tr>
<tr>
<td>Churchill et al. [9]</td>
<td>$K_a = 186 (SU)^{0.5} H^{1.0}$</td>
</tr>
<tr>
<td>Owens et al. [16]</td>
<td>$K_a = 24.9 (1 + Fr^{0.5}) U H^{1.0}$</td>
</tr>
<tr>
<td>Langbein and Durum [17]</td>
<td>$K_a = 3 (1 + 0.17 Fr^{3})(SU)^{0.375} H^{1.0}$</td>
</tr>
<tr>
<td>J.D.Parkhurst &amp; R. D. Pomeroy et al. [20]</td>
<td>$K_a = 31,200 SU$ for $Q &lt; 0.28$ m$^3$/s</td>
</tr>
<tr>
<td>Smoot [21]</td>
<td>$K_a = 15200 SU$ for $Q &gt; 0.28$ m$^3$/s</td>
</tr>
<tr>
<td>Moog [22]</td>
<td>$K_a = 543 S^{0.5236} U^{0.532} H^{0.4}$</td>
</tr>
<tr>
<td>Jha et al. [23]</td>
<td>$K_a = 1740 U^{0.46} S^{0.7} H^{0.74}$ for $S &lt; 0.00$</td>
</tr>
<tr>
<td>Jha et al. [24]</td>
<td>$K_a = 5.791 U^{0.50} H^{0.20}$</td>
</tr>
<tr>
<td>Jha et al. [24]</td>
<td>$K_a = 0.603286 U^{0.4} S^{1} H^{0.124}$ for $Fr &lt; 1$</td>
</tr>
</tbody>
</table>

Where $U =$ velocity in meter per sec, $H =$ depth of waterway, $S =$ stream bed slope, $Fr =$ Froude number

Fig. 1. location of river Tungabhadra selected for the study
Table 2. Ka values measured by DOBT for different flow condition of river Tungabhadra

<table>
<thead>
<tr>
<th>Sl. No</th>
<th>Velocity</th>
<th>Depth</th>
<th>BED slope</th>
<th>Flow</th>
<th>Ka</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>m/s</td>
<td>in m</td>
<td></td>
<td>m3/s</td>
<td></td>
</tr>
<tr>
<td>January</td>
<td>0.51</td>
<td>0.22</td>
<td>0.0024</td>
<td>0.71</td>
<td>6.35</td>
</tr>
<tr>
<td>February</td>
<td>0.59</td>
<td>0.35</td>
<td>0.0024</td>
<td>0.64</td>
<td>6.1</td>
</tr>
<tr>
<td>March</td>
<td>0.62</td>
<td>0.28</td>
<td>0.0024</td>
<td>0.56</td>
<td>6.577</td>
</tr>
<tr>
<td>April</td>
<td>0.71</td>
<td>0.37</td>
<td>0.0024</td>
<td>0.548</td>
<td>6.6</td>
</tr>
<tr>
<td>May</td>
<td>0.73</td>
<td>0.5</td>
<td>0.0024</td>
<td>0.49</td>
<td>5.9</td>
</tr>
<tr>
<td>June</td>
<td>0.55</td>
<td>0.55</td>
<td>0.0024</td>
<td>0.497</td>
<td>6.57</td>
</tr>
<tr>
<td>July</td>
<td>0.56</td>
<td>0.6</td>
<td>0.0024</td>
<td>0.52</td>
<td>6.0794</td>
</tr>
<tr>
<td>October</td>
<td>0.57</td>
<td>0.61</td>
<td>0.0024</td>
<td>0.292</td>
<td>6.167</td>
</tr>
<tr>
<td>November</td>
<td>0.59</td>
<td>0.64</td>
<td>0.0024</td>
<td>0.46</td>
<td>4.1</td>
</tr>
<tr>
<td>December</td>
<td>0.6</td>
<td>0.66</td>
<td>0.0024</td>
<td>0.52</td>
<td>5.9</td>
</tr>
</tbody>
</table>

Fig. 2. Segmentation scheme to calculate Ka River

Fig. 3. Standard error and mean multiplicative error for predictive equation
Fig. 4. Normal mean error for predictive equation

Fig. 5. Coefficient of correlation r value for predictive equation

Fig. 6. Comparison of predicted and measured $K_a$ Values
Evaluating and assessment error in predicted re-aeration equation

The performance of thirteen most popular equations are shown in Table 1 which have been evaluated by using statistical error method by application standard error, normal mean error method, mean multiplicative error method and correlation coefficient (r). The backdrop of this method is related to Jha [2001]. The Standard error and Normal mean error method are differential error which are calculated by using following equations.

\[ SE = \sqrt{RT \left( \sum_{i=1}^{N} \left( \frac{(Kp - Km)^2}{Km} \right) / N \right)} \]  (6)

\[ NME = \left( 100\% / \sum_{i=1}^{N} \left( \frac{(Kp - Km)^2}{Km} \right) / Km \right) \]  (7)

where N= number of reaerations, Kp=predicted value, Km=measured value.

The mean multiplicative error method used to analysis result for estimation of impact inaccuracy [Moorg and Jirka].

\[ MME = \left( \frac{1}{N} \left( \ln(Kp-Km)\right)^2 \right) / N \]  (8)

Correlation of coefficient (r) can be computed by pearson method. In this method we can check fractional variance (r) which must closer to unity which shows better result.

\[ r = \left( 1 - S_{xy}/S_{yy} \right)^{0.5} \]  (9)

where \( S_{xy} = \) sum of square of difference between the observed and computed values and \( S_{yy} = \) sum of squares of departures of observed values of Kp from the mean of observed values.

ANALYSIS OF THE RESULT

Utilizing the measured information from the field survey i.e., stream velocity, flow, water depth, slope and along with physical-chemical and biological water quality parameter, information index from river Tungabhadra, reaeration rate coefficient (Ka) value were calculated for all thirteen reaeration equation which shown in Table 1.

Fig. 3 represents the graph of Mean Multiplicative Errors and Standard Errors (MME & SE) for Equations (6) and Equation (8). The chart in Fig. 4 represents the MME computed for Equation (7). On the other hand, the chart represented by Fig. 5 shows represents the correlation coefficient r for the whole predictive equations.

It has been seen that the reaeration coefficient generated by Jha et al. [23] demonstrates the best concurrence with estimated values as far as SE, NME and MME ( SE=1.24, NME=0.159, MME=2.34) trailed by the condition created by Moog and jirk [22] (SE = 1.49, MME = 0.18, NME = 13.8). Notwithstanding, as far as correlation coefficient, the condition created by Jha et al. [23] demonstrates better assertion (r = 0.90) than that the condition of Moog and jirk [22] (r = 0.45).


The Standard Error, Multiplicative mean error and Normal Mean error values obtained by the conditions proposed by Langbein and Durum's [17] (SE = 1.56, MME = 3.156 and NME = -0.096) are likewise nearer to the estimations of Jha et al. [23] and Cadwallader and McDonnell, [18] (SE = 1.81, MME = 3.74, NME = 0.264). For O'Connor and Dobbins [7] condition (SE = 2.53, MME = 4.0519 and NME = 0.031), Parkhurst and Pomeroy, [20] yet the correlation coefficient is low (r = 0.4). The estimated errors for the conditions proposed by different authors, specifically Churchill et al. [9] Tsivoglou and Wallace, [13] Krenkel and Orlob, [15] Owens et al. [16] Smoot, [21] and Thackston and Krenkel [19] are in fractional concurrence with the observed qualities.

As the Standard Error and Normal Mean error values give differential-error [23], the outcomes are viewed as one-sided for bigger qualities and other side file down mistakes. The MME value, which utilizes the proportion of anticipated and estimated values, is viewed as most exact
3. CONCLUSION

The results derived from the popular predictive equation for reaeration constants found in the literature were analysed using error estimation statistical methods. The results of the error estimations proved that the equation proposed by Jha et al. [23] has the highest correlation with the values measured for $ka$. On the other hand, equations proposed Jha et al. [24] that make use of the slope parameters resulted in values that are significantly higher than the measured values. Other equations analysed with the statistical error estimation method are those proposed by Langbein and Durum [17], Moog and Jirka [22], and Parkhurst and Pomeroy, [20]. And they all showed high correlation with the measured values.

It is essential to note that the results derived from the enhanced reaeration equation that was designed for river Tungabhadra has higher levels of accuracy. It is recommended for use for future extensive field research in the river. The enhanced predictive reaeration equation that was created for River Tungabhadra is also valuable for other streams that have similar climatic, geographical, and hydraulic conditions.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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